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The primary function of the Astropulse program is to dedisperse potential pulsed signals, then determine whether the dedispersed pulse surpasses an appropriate threshold. I will discuss the theory behind dedispersion, the logic of the algorithm, and then methods we use to select the thresholds. Then I will calculate the sensitivity of Astropulse in Jy $\mu$s.

### 3.2. Dedispersion

Between a radio pulse’s source (i.e. black hole, pulsar, or ET) and our detector, the pulse must travel through the Interstellar Medium (ISM). The ISM is composed of neutral hydrogen and helium atoms which have negligible effect on the wave’s propagation, as well as a plasma component consisting of protons, other positively charged ions, and free electrons. As the wave travels through the plasma, it’s affected by dispersion, in a manner analogous to the dispersion of light in a prism. In this case, the high frequency component moves slightly faster through the ISM than the low frequency component. The time delay between any two frequencies is given by Wilson et al. (2009):

$$\Delta \tau = \frac{e^2}{2 \pi cm_e} \left( \frac{1}{\nu_1^2} - \frac{1}{\nu_2^2} \right) \int N(\ell) d\ell$$

(1)

Where $N(\ell)$ is the electron number density at position $\ell$ along the pulse’s path. Then $\int N(\ell) d\ell$ is the dispersion measure, or DM, and depends only on the distribution of plasma between the source and detector, not on the frequency of the radiation. If $N(\ell)$ is measured in electrons cm$^{-3}$, and $\ell$ is measured in parsecs, we say that the dispersion measure DM is measured in pc cm$^{-3}$. A useful approximation for the dispersion measure weighted mean electron density in our Galaxy is $N = 0.03$ cm$^{-3}$ (Guélin 1973).

Consider, for instance, the Crab pulsar, at DM = 56.791. Astropulse sees a bandwidth of 2.5 MHz centered at 1420 GHz. We can deduce an approximate $\Delta \tau$ by saying that $\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2} \approx \frac{2}{\nu^3} \Delta \nu$, setting $\Delta \nu = 2.5$ MHz and $\nu = 1420$ GHz. The resulting $\Delta \tau = 0.00041145$ s, which differs from the exact result by 0.000155%. Astropulse has 0.4 $\mu$s samples, so this is 1028.6 samples, for a ratio of samples / DM = 18.11. Alternatively,

$$\Delta \tau = (8.3 \ \mu s) \frac{\Delta \nu(MHz)}{\nu^3(GHz)} \cdot DM(pc \ cm^{-3})$$

(2)

So a hypothetical Crab pulse that initially would have been concentrated in one time sample of our measured time series will be dispersed to $N = 1029$ time samples. This means it will be submerged in $N$ times as much noise (on average), if the noise is measured in Jy $\mu$s. (The noise in Jy $\mu$s increases when we integrate the noise over a longer duration.)
If instead we measured the noise in Jy, there would be no time integral.) The probability density function (pdf) of the noise (measured in Jy $\mu$s) is an incomplete gamma function, as discussed below. As the number of samples $N$ increases, this pdf approaches a normal distribution, with standard deviation proportional to $\sqrt{N}$. The signal is not detectable unless it is substantially stronger than this $\sqrt{N}$, because otherwise it would look like a fluctuation in the noise. For a larger dispersion measure, the problem would be even worse.

Therefore, we have to reconstruct the original pulse, bringing together the component frequencies and reassembling them so that the signal takes up a single time sample again. This will prevent it from being buried in noise, allowing us to improve our sensitivity by lowering our detection threshold.

We can depict the dedispersion process in a time vs. frequency plot. Note that there is no strictly correct mathematical way of depicting a function’s time vs. frequency, due to the uncertainty principle. That is, we cannot write an arbitrary amplitude $A(t)$ in the form $\nu(t)$. For instance, consider a signal whose time vs. amplitude graph looks like a delta function. It is supposed to contain all frequencies ... so what frequency does it have at time 0? At times other than 0? A time vs. frequency plot of this sort is meaningful only to the extent that the signal looks locally like a monochromatic (sine) wave.

Instead, we can perform fourier transforms on groups of consecutive samples. Say we FFT 64 samples. They are located at some time $t$, although the resolution is now 64 sample widths. The FFT gives us an amplitude (hence a power) for each of 64 frequencies, so we now have a plot with $z =$ power, $y =$ frequency, and $x =$ time. Such a plot is shown in Figure 1, in the extreme case that a single frequency is present at each time for the dispersed signal. (I.e. there is no noise.) In this plot, the power is represented by the darkness of each pixel.

![Fig. 1.— The diagonal line represents the initial, dispersed signal. The vertical line is the dedispersed signal. X axis is time in Astropulse samples, and Y axis is frequency in MHz. The pulse has a dispersion measure (DM) of +50 pc cm$^{-3}$.](image_url)

We have two choices for our methodology: coherent dedispersion and incoherent dedispersion. Astropulse uses coherent dedispersion, whereas other radio surveys use incoherent dedispersion. Incoherent dedispersion is much more computationally efficient,
and for longer timescales it’s almost as good as coherent dedispersion. However, as we will see, Astropulse would be unable to examine the 0.4 μs timescale without coherent dedispersion.

Incoherent dedispersion means that the signal’s power spectrum is calculated, and the power vs. time of each sub-band is analyzed. The method is called “incoherent” for this reason – the phase information about individual frequencies is lost; only the total power of each subband is retained. Then, the sub-bands are realigned at all possible dispersion measures, in an effort to find one DM at which the components align to produce a large power. Suppose we use Δt to denote the difference between the time delay of the highest and lowest frequencies in our bandwidth. Then the sub-band with frequency ν₀ + ν (where ν₀ is the center frequency) is shifted by a time νΔν Δt. However, incoherent dedispersion is limited in two ways. First, the goal of recording power vs. time makes sense only on a timescale greater than 1/dν, where dν is the width of each sub-band. This is because of time-frequency uncertainty. Second, in each sub-band the pulse is dispersed by Δτ dν Δν, where (as above) Δτ is the time over which the pulse is dispersed in the whole band. So the method cannot localize the pulse better than this. Combining these two limits, we find that the minimum timescale for incoherent dedispersion, dt, happens when:

\[\Delta \tau \cdot \frac{d
u}{\Delta \nu} = \frac{1}{d
u} \quad (3)\]

\[d
u^2 = \frac{\Delta \nu}{\Delta \tau} \quad (4)\]

\[d
u = \sqrt{\frac{\Delta \nu}{\Delta \tau}} \quad (5)\]

\[dt = \sqrt{\frac{\Delta \tau}{\Delta \nu}} \quad (6)\]

\[= \sqrt{\frac{411 \text{ μs}}{56.791}} \cdot \left(\frac{1.42 \text{ GHz}}{\nu_0}\right)^3\left(\frac{\Delta \nu}{2.5 \text{ MHz}}\right) \cdot (\Delta \nu)^{-1} \quad (7)\]

\[= 12.8 \text{ μs} \left(\frac{DM}{56.791}\right)^{0.5}\left(\frac{\nu_0}{1.42 \text{ GHz}}\right)^{-1.5} \quad (8)\]

For the Crab pulsar, this is a limit of 12.8 μs, or 32 samples. For a more distant source, the limit might be as much as 50 μs, or 124 samples. (Astropulse considers sources with a DM as high as 830 pc cm⁻³.)

Coherent dedispersion is an alternative technique that allows better time resolution. Coherent dedispersion deals with amplitude rather than power, preserving phase information. If \(F(n)\) is the original pulse as a function of sample number \(n\), then suppose \(D[F]\) is the dispersed pulse. We can show that \(D\) is just a convolution. In particular, \(D[F] = F \ast D[\delta]\), where \(*\) is convolution and \(\delta\) is the discrete δ function.

We know that \(D\) is time translation-invariant and linear, because Maxwell’s equations in a plasma have these properties. That is, if we combine Maxwell’s equations with:

\[m_e \ddot{v} = -eE \quad (9)\]
\[ \vec{J} = -n_e \vec{v} \quad (10) \]

for the velocity of the electrons (assuming the protons do not accelerate substantially), we have a linear set of equations in \( \rho, \vec{J}, \vec{E}, \vec{B}, \) and \( \vec{v} \). For example, figure 2 depicts a dispersed (chirped) delta function:

![Delta function and chirped delta function](image)

**Fig. 2.** Delta function and chirped delta function. The x axis is time, and the y axis is amplitude. The units are not meaningful. Note that the chirped function tapers off toward 0 amplitude at high and low times. This is because the initial function is not a true delta function; it has nonzero width. For an infinitely narrow delta function, the chirped function’s envelope would not taper off at all.

When the delta function is time translated, so is the dispersed version (figure 3.)

And when two delta functions are summed, so are the dispersed versions (figure 4.)

From figures 3 and 4, we conclude that dispersion is linear and time translation invariant.

Then:

\[ F(n) = \sum_{n'} F(n') \delta(n - n') \equiv (F \ast \delta)(n) \quad (11) \]
Fig. 3.— Time translated chirped delta functions. The x axis is time, and the y axis is amplitude. The units are not meaningful.

\[ D[F](n) = \sum_{n'} F(n') \cdot D[\delta](n - n') \equiv (F \ast D[\delta])(n) \] (12)

where the last step is by linearity and time translation invariance of \( D \).

Now the pulse that arrives at our detector is the output pulse \( F \ast D[\delta] \), and we want to determine \( F \). To do this, we use the fact that the convolution operator is related to the Discrete Fourier Transform (DFT). We have \( \text{DFT}(f \ast g) = \text{DFT}(f) \cdot \text{DFT}(g) \)

Therefore

\[ \text{DFT}(D[F]) = \text{DFT}(F \ast D[\delta]) = \text{DFT}(F) \cdot \text{DFT}(D[\delta]) \] (13)

\[ F = \text{DFT}^{-1}\left( \frac{\text{DFT}(D[F])}{\text{DFT}(D[\delta])} \right) \] (15)

Convolution “the slow way” takes time \( O(N^2) \) – that is, convolution by multiplying every value of one function by every value of another function, then adding up the results.
But the FFT is faster, taking time $O(N \log N)$, where $N$ is the length of the FFT. (“DFT” refers to a mathematical function, and “FFT” refers to a specific type of algorithm that computes that function quickly.) For every $N$ samples, coherent dedispersion takes time $O(M N \log N)$, where $M$ is the number of dispersion measures to test. If $L$ is the number of samples in the data stream, coherent dedispersion takes time $O(M L \log N)$.

On the other hand, incoherent dedispersion tests $M/n$ dispersion measures, where $n$ is the time resolution (in samples) implied by the sub-band width. This time resolution is limited by time-frequency uncertainty as well as dispersion within sub-bands, and is dependent on the particular value of the DM. It can be calculated from (Equation 8). For example, with a resolution of 20 $\mu$s and a sample duration of 0.4 $\mu$s, $n = 50$. For each dispersion measure, we must process $L$ samples, so we require time $O(M L/n)$. (Some additional time is required to FFT the data into a power spectrum, but this process is not dominant.) So the time ratio between coherent and incoherent dedispersion is $O(n \log N)$. In our case, $\log N = 15$, so this goes like $50 \cdot 15 = 750$, a large ratio. Of course we can’t calculate the ratio exactly, without knowing the respective constant factors. But this helps explain why coherent dedispersion is not generally used for surveys.

To find the chirp function $\text{DFT}(D[\delta])$, we must first find $D[\delta]$. This is just the chirped
Delta function. Note that the arrival time of frequency $\nu$ is given by Equation 1 as:

$$t(\nu) = A\frac{\nu^2}{\nu^2} + t_0$$  \hfill (16)

Thus, for interstellar dispersion, we expect higher frequencies to arrive first, which agrees with Equation 16 for $A > 0$. We want $\nu = 1420$ MHz to arrive at time 0, which requires that $t_0 < 0$.

Now let’s determine $\nu(t)$, the frequency arriving at time $t$. This has:

$$t = A\frac{\nu^2}{\nu^2} + t_0$$  \hfill (17)

$$t - t_0 = A\frac{\nu^2}{\nu^2}$$  \hfill (18)

$$A\nu^{-2} = t - t_0$$  \hfill (19)

$$\nu^{-2} = \frac{t - t_0}{A}$$  \hfill (20)

$$(\nu^{-2})^{-1/2} = \left(\frac{t - t_0}{A}\right)^{-1/2}$$  \hfill (21)

$$\nu(t) = A^{1/2}(t - t_0)^{-1/2}$$  \hfill (22)

Then

$$f(t) = \exp(2\pi i \int_0^t \nu(t')dt')$$  \hfill (23)

is the pulse amplitude $D[\delta]$ as a function of time, if $\infty$ frequency arrives at time $t_0$. We want to find the chirp function $\text{FFT}(D[\delta]) = \tilde{f}(\nu)$, so we can divide by it, as in Equation 13. We have

$$\tilde{f}(\nu) = \int_{-\infty}^{\infty} f(t) \exp(-2\pi it\nu)dt$$  \hfill (24)

$$= \int_{-\infty}^{\infty} \exp(2\pi i(\int_0^t \nu(t')dt' - \nu t))dt$$  \hfill (25)

We evaluate this integral using the method of steepest descent, which is used to calculate an integral of the form

$$\int_{-\infty}^{\infty} e^{g(t)}dt$$  \hfill (26)

The method is typically considered valid if $\dot{g} = 0$ at some $t = \tilde{t}$, and the path of integration can be arranged so that $\Re(g)$, the real part of $g$, has a global maximum (along
the path) at \( \tilde{t} \). Also, the third and subsequent derivatives of \( g \) should be small enough that \( g \) can be treated as quadratic, at least until \( \Re(g) \) becomes much smaller than its maximum. Then \( e^{g(t)} \) can be treated approximately as a Gaussian.

In our case \( g \) is pure imaginary along the path of integration, leading to an oscillating complex exponential term. So we cannot have \( \Re(g) \) smaller than the maximum, although it is conceivable that we could achieve this by changing the path of integration. But instead, let’s show that \( e^{g(t)} \) can be treated as a Gaussian anyway. The right and left “tails” can be ignored; for instance, what is \( \int_{t_1}^{\infty} \cos(g(t))dt \) where \( g(t) \) and \( g'(t) \) are monotonically increasing? We can change variables like \( dt = t' dg \) to get \( \int_{g_1}^{\infty} \cos(g(t))t'(g)dg \). Then \( t'(g) \) is monotonically decreasing, so that each half-period integral of \( \cos(g) \) is smaller than the previous. Since these integrals alternate in sign, the whole integral is on the order of \( t'(g_1) = 1/g'(t_1) = \omega_1 \), if \( \omega_1 \) is the initial frequency.

Roughly, we want \( 1/g'(t) \ll \sqrt{2 \lambda / \nu} \) (see Equation 34) by the time any derivatives matter that are higher order than \( g''(t) \). In our case,

\[
\begin{align*}
g(t) &= 2\pi i(\int_0^t \nu(t')dt' - \nu t) \quad (27) \\
g'(t) &= 2\pi i(\nu(t) - \nu) = 2\pi i(A^{1/2}(t - t_0)^{-1/2} - \nu) \quad (28) \\
g''(t) &= -\pi iA^{1/2}(t - t_0)^{-3/2} \quad (29) \\
g'''(t) &= \frac{3}{2}\pi iA^{1/2}(t - t_0)^{-5/2} \quad (30)
\end{align*}
\]

The \( g'''(t) \) term in the Taylor series expansion is insignificant as long as

\[
|\frac{1}{4}\pi A^{1/2}(\tilde{t} - t_0)^{-5/2}(t - \tilde{t})^3| \ll 1
\]

Then

\[
\nu = \nu(\tilde{t}) = A^{1/2}(\tilde{t} - t_0)^{-1/2}
\]

so we require

\[
|\frac{1}{4}\pi \nu^5 A^{-2}(t - \tilde{t})^3| \ll 1 \\
|t - \tilde{t}| \ll \left(\frac{4}{\pi}\right)^{1/3} \nu^{-5/3} A^{2/3}
\]

\[
A = 4 \cdot 10^{15} \text{ Hz} \cdot \text{ DM (pc cm}^{-3}\text{)}, \text{ so for a DM of 200 pc cm}^{-3}\text{, this last RHS is on the order of } 5 \cdot 10^{-4} \text{ s.}
\]

We want to know whether \( 1/g'(t) \ll \sqrt{2 \lambda / \nu} \) for all points outside this region. Then \( g'(t) = 2\pi(A^{1/2}((t - \tilde{t}) + (\tilde{t} - t_0))^{-1/2} - \nu) \), and \( t - \tilde{t} = \epsilon \) is small compared with \( t_0 = -0.41 \) s. Now, define
\[ \nu_0 = \nu(0) = A^{1/2}(-t_0)^{-1/2} = 1420 \text{ MHz} \quad (32) \]

Then \( \nu \approx \nu_0 \), so that \( \tilde{t} = t(\nu) \approx 0 \) and therefore \( \tilde{t} - t_0 \approx -t_0 \). This means that \( \epsilon \) is also small compared with \( \tilde{t} - t_0 \). So

\[
g'(t) = 2\pi A^{1/2}(\tilde{t} - t_0)^{-1/2}(1 - \frac{1}{2} \frac{\epsilon}{t - t_0} - 1)
\]
\[
= -\pi A^{1/2} \frac{\epsilon}{(\tilde{t} - t_0)^{3/2}}
\]
\[
= -\pi \frac{\epsilon \nu^3}{A}
\]
\[
1/|g'(t)| = 0.9 \cdot 10^{-9}
\]

Furthermore,

\[
\sqrt{\frac{2A}{\nu^3}} = 1.7 \cdot 10^{-6} \quad (33)
\]

So \( 1/g'(t) \ll \sqrt{\frac{2A}{\nu^3}} \) as desired. We are probably justified in applying the method of steepest descent. (Although this argument by no means constitutes a proof.) The method of steepest descent says that the value of the integral is:

\[
\pm \sqrt{\frac{2\pi}{|g''(t)|}} \exp(g(\tilde{t})) \quad (34)
\]

The factor in front is just \( \pm \sqrt{\frac{2\pi}{|g'(t)|}} = \pm \sqrt{\frac{2A}{\nu^3}} \) using Equations 29 and 31. Then we consider the next factor,

\[
g(\tilde{t}) = 2\pi i \int_0^\tilde{t} \nu(t') dt' - \nu \tilde{t}
\]
\[
\nu(t') = A^{1/2}(t - t_0)^{-1/2}
\]
\[
\int_0^\tilde{t} \nu(t') = A^{1/2}2(t - t_0)^{1/2}\tilde{t}_0
\]
\[
\tilde{t} = \frac{A}{\nu^2} + t_0
\]
\[
\int_0^\tilde{t} \nu(t') = A^{1/2} \cdot 2(\frac{A}{\nu^2})^{1/2} - 2A^{1/2}(-t_0)^{1/2}
\]
\[
= 2A^{1/2}(\frac{A^{1/2}}{\nu} - (-t_0)^{1/2})
\]
\[ g(\tilde{t}) = 2\pi i(2\sqrt{A\left[\frac{\sqrt{A}}{\nu} - (-t_0)^{1/2}\right]} - \nu|\frac{A}{\nu^2} + t_0|) \]
\[ = 2\pi i(2\frac{A}{\nu} - 2\sqrt{A}(-t_0)^{1/2} - \frac{A}{\nu} - \nu t_0) \]
\[ = 2\pi i\left(\frac{A}{\nu} - 2\sqrt{A}(-t_0)^{1/2} - \nu t_0\right) \]

Then, using 32:

\[ (-t_0)^{-1/2} = A^{-1/2}\nu_0 \]
\[ -t_0 = A\nu_0^{-2} \]
\[ t_0 = -A\nu_0^{-2} \]
\[ g(\tilde{t}) = 2\pi i\left(\frac{A}{\nu} - 2\sqrt{A}\sqrt{A}\nu_0^{-1} + \nu A\nu_0^{-2}\right) \]
\[ = 2\pi iA\left(\frac{1}{\nu} - \frac{2}{\nu_0} + \nu \frac{\nu_0}{\nu_0^2}\right) \]
\[ = 2\pi iA\left(\frac{\nu_0^2 - 2\nu\nu_0 + \nu^2}{\nu\nu_0^2}\right) \]
\[ = 2\pi iA\frac{(\nu - \nu_0)^2}{\nu\nu_0^2} \]

Since \( \nu \approx \nu_0 \) in our application, the chirp function looks something like \( \exp(2\pi iA\frac{(\nu - \nu_0)^2}{\nu\nu_0^2}) \), and the exponent would be a quadratic. But the extra factor \( \nu_0/\nu \) is easily included in our dedispersion algorithm, so we have done so. This extra factor gives rise to a curvature in the time vs. frequency plot of the dispersed pulse, which must be on the order of \( (\nu - \nu_0)/\nu_0 \), or in our case \( 1.25 \text{ MHz}/1.42 \text{ GHz} = 0.00088 \), which is about 1 sample out of every 1000. Therefore, this curvature could be quite significant for small coadds, especially for large DM.

### 3.3. Algorithm Logic

Astropulse loops through the data at several nested levels. We consider DMs ranging from \( \pm 49.5 \text{ pc cm}^{-3} \) to \( \pm 830 \text{ pc cm}^{-3} \):

1. Large dm chunk: blocks of 128 dms at a time.
   
   At the end of each large dm chunk loop, Astropulse runs the fast folding algorithm on the entire data set, with a resolution of 128 samples.

1.a Small dm chunk: blocks of 16 dms at a time.
   
   At the end of each small dm chunk loop, Astropulse runs the fast folding algorithm on the first \( 1048576 = 2^{20} \) samples of data, with a resolution of 16 samples.
2. Data chunks of size $32768 = 2^{15}$ samples.

Astropulse increments the data chunk start point by $16384 = 2^{14}$ samples on each pass through this loop. This redundancy is necessary because otherwise some pulses might extend beyond the edge of a data chunk. We want to operate on the whole pulse with a single Fourier transform.

Within this loop, we compute the Fast Fourier Transform (FFT) of the data for use in convolution, producing a frequency spectrum that will later be dechirped and then FFT’ed in reverse.

3. All dms within a small dm chunk.

4. Sign of the dm (positive and negative).

We consider negative dms for a few reasons:

(a) Extraterrestrial civilizations might communicate using signals dispersed with negative dms.

(b) Signals detected at both positive and negative dm can be a sign of radar or other radio frequency interference (RFI).

5. Scales (or “co-adds”) $0 - 9$.

Scale (or co-add level) $\ell$ means that the client combines $2^\ell$ adjacent samples and measures the total power.

6. Samples within the data chunk

If the power in $2^\ell$ samples is above a certain threshold, then Astropulse reports a pulse.

### 3.3.1. The Fast Folding Algorithm

To search for periodic pulses, we employ the the Fast Folding Algorithm (FFA) (Staelin 1969). Before folding, we collapse our data in the time domain by summing either 16 or 128 samples, depending on whether it was created during the large or small dm chunk loop. Below, a collapsed set of samples will be called a “bin.”

0. Loop over sub buffers

Currently, we only have one “sub buffer”, which consists of the entire collapsed time series or some fraction thereof. Astropulse operates the FFA on the entire workunit of $2^{25}$ samples, or else on $2^{20}$ samples. In the future, we have the option to divide the workunit up into smaller sub buffers, because (it turns out, see below) the run time of the FFA goes like $N^2$. This means that by dividing the workunit in half, we would save a factor of 2 in run time, $N^2 \rightarrow \left(\frac{N}{2}\right)^2 + \left(\frac{N}{2}\right)^2 = \frac{N^2}{2}$. However, this modification would also degrade our sensitivity by $\sqrt{2}$, see Section 3.5, so we have a strong preference to run the FFA on the entire collapsed time series. Furthermore, examining 2 short sub buffers is not much better than examining one short sub buffer, because a repeating signal could be present in all of the data from a given beam transit time. (Hence it would be detectable in either sub buffer.) So we opted instead to reduce the total length of the fold buffer in the case of dm size 16.
1. **Loop over frequencies**

   We define the frequency to be the number of periods over the entire sub buffer. The smallest frequency in this loop is always $F_{\text{min}} = 137$, and the largest frequency is twice that. The frequency increments via multiplication by $(1 + \frac{1}{N})$, where $N$ is the number of bins in the sub buffer.

   We construct a new, shorter time series from the original time series, by folding it. That is, we divide the time series into $F$ chunks, where $F$ is the frequency, and add them up.

2. **Loop over subfrequencies**

   This part is what makes the folding “fast.” Instead of re-folding at all frequencies greater than $2F_{\text{min}}$, we simply fold frequency $F$ in half to make $2F$, again to make $4F$, and so on.

3. **Loop over coadds**

   As in the single pulse algorithm, we search for pulses that take up more than one bin. In this case, of course, a single bin could be 128 samples, two bins would be 256 samples, etc.

4. **Loop over bins**

   Examine each bin to see if the power exceeds the threshold, see 3.4.

   We can compute the dependence of the run time on $N$, the number of bins, as follows.

   1. We must search $N$ times as many frequencies, since the frequencies differ by the ratio $(1 + \frac{1}{N})$.

   2. We must search $N$ times as many bins (or phases of the repeating signal) for each frequency.

   This makes an overall factor of $N^2$. The run time is also influenced by the number of times we run the Fast Folding Algorithm. (E.g. once every 128 DMs or every 16 DMs.)

3.4. **Thresholds**

   Astropulse searches for pulses whose power exceeds certain thresholds. These thresholds can be calculated either experimentally or theoretically. I’ll start by finding the theoretical values, then point out some of the uncontrollable factors that make these values inaccurate, and finally I’ll describe the experimental methods for calculating thresholds.

   **3.4.1. Single pulse thresholds: theory**

   We want to calculate the distribution (pdf) of the noise power per $2^\ell$ samples, after dechirping.
First, we assume that the pre-deconvolution time series is pure white noise; that is, each bit of a two bit complex sample is independently distributed with equal probability of a 0 or 1, so each $f(t)$ has equal probability for $\pm 1 \pm i$. Then we deconvolve this data by FFT. In other words,

$$\tilde{f}(k) = \frac{1}{\sqrt{N}} \sum_{t=0}^{N-1} f(t)e^{-2\pi ikt/N}$$

The distribution of a single $\tilde{f}(k)$ is Gaussian (by the law of large numbers), and the variance of $\Re(\tilde{f}(k))$ comes from the sum of the $2N$ variances of the $\Re(f(t))$ and $\Im(f(t))$, or $2N(\frac{1}{\sqrt{N}})^2E(\sin^2) = 1$. We will then multiply by a chirp function which looks like $e^{i\text{phase}(k)}$, and doesn’t affect the individual variances. Finally, we run the inverse FFT. We assume that the chirp function has scrambled the phases, so that the result is once again a sum of independent and identically distributed (iid) random variables. The previous argument applies, and since the variances of $\Re(\tilde{f}(k))$ and $\Re(f(t))$ are both 1, we get the same result as before: a standard Gaussian, $\sigma = 1$.

The power in each sample after deconvolution will be distributed like $|\Re(\tilde{f}(k))|^2 + |\Im(\tilde{f}(k))|^2$, the sum of the squares of two standard Gaussians. This distribution is easily calculated; the joint distribution is:

$$\frac{1}{\sqrt{2\pi}} e^{\frac{x^2}{2}} \cdot \frac{1}{\sqrt{2\pi}} e^{\frac{y^2}{2}} dxdy$$

$$= \frac{1}{2\pi} e^{-\frac{u}{2}} rdrd\theta$$

$$\to e^{-\frac{r^2}{2}} rdr$$

$$= e^{-u}du$$

where $u = \frac{x^2+y^2}{2}$ is half the power in one sample, in the time domain. Therefore, half the power is exponentially distributed with mean 1; or equivalently, the power is exponentially distributed with mean 2.

In future calculations, we will normalize to half of this power, so that the average power per sample is 1.

So for instance, if after dechirping we find that a certain sample has a power of 15.3, we conclude that only one in $e^{15.3}$ samples has a comparable power. To ascertain how unlikely this is, we need to calculate how many such samples we have examined over the entire course of the experiment. This would be:

$$7 \text{ beams} \times 2 \text{ polarizations} \times \frac{1}{3} \text{ on fraction} \times 3 \text{ years}$$

$$\times 1 \text{ workunit per } 13 s \times 2^{25} \text{ samples per workunit} \times 14208 \text{ dms}$$

$$\times 2 \text{ dm signs} \times 10 \text{ coadds}$$

$$= 3.24 \times 10^{20} = e^{47.2}$$

With a threshold of 47.2, we would rule out all but one noise pulse over the course of
an entire year. However, it seems more prudent to allow one noise pulse per workunit, and sort out false pulses later.

In this case, we just want:

$$2^{25} \text{ samples per workunit} \times 14208 \text{ dms} \times 2 \text{ signs} \times 10 \text{ coadds} = e^{29.9} \quad (40)$$

When we collapse $n = 2^\ell$ samples to make one bin, we are adding up that many exponential distributions. The result is a gamma distribution, with scale parameter 1 and shape parameter $n$. The pdf is

$$\frac{1}{\Gamma(n)} x^{n-1} e^{-x} \quad (41)$$

and the cumulative distribution function is

$$\frac{\gamma(n, x)}{\Gamma(n)} \quad (42)$$

where $\gamma$ is the incomplete gamma function. The first few pdfs look like figure 5.

3.4.2. Expected discrepancies with the model

A few differences from the model can be expected:

1. **Hydrogen line and filter shape.** We assumed above that the input data is white noise. In practice, this is not the case, because a portion of our bandwidth has higher power due to the hyperfine Hydrogen line. The strength of this line can vary depending on our RA and dec. Then $\hat{f}(k)$ no longer have equal standard deviations. This will cause some correlation between the deconvolved power of adjacent samples, which will modify the pdf of the binned power, increasing the probability of larger powers.

To see this, consider the simplest, most extreme case, where the Hydrogen line is a strong delta function of amplitude $A$ at frequency $k_0$, where $A$ is distributed randomly like a Gaussian with standard deviation $\sigma$. Then if $\hat{f}$ is the dechirped amplitude, $\hat{f}(t) = A e^{2\pi i k_0 t / N}$. So $|\hat{f}(t)|^2$ is exponential with power $\sigma^2$. But $|\hat{f}(t_1) + \hat{f}(t_2)|$ is $A |e^{2\pi i k_0 t_1 / N} + e^{2\pi i k_0 t_2 / N}|$. If $t_1 \sim t_2$, this is roughly $2A$, which is Gaussian with standard deviation $2\sigma$ and variance $4\sigma^2$. Whereas if we binned samples by adding iid exponentials, the variances would simply add to give $2\sigma^2$. So the Hydrogen line increases the variance.

In actuality, the effect of the Hydrogen line is not so pronounced, but the idea is similar. In the same way, the nonuniform shape of our low pass filters also causes the signal to differ from white noise.

2. **Other disparities**
Fig. 5.— Gamma distributions. The x axis is power (divided by 2, as per our convention), and the y axis is probability per unit power. The leftmost distribution, which is exponential, belongs to $n = 1$. The rest are $n = 2, 4, 8, 16, 32, 64$. Notice that the rightmost distribution is nearly a normal distribution.

Even in the absence of the Hydrogen line, tests reveal other differences between the theoretical and actual distributions. For larger bin sizes, the variance is less than expected.

It’s easy to see that power per sample cannot be independently distributed, even in the case of white noise. This is because the total power over all samples must be a constant; in our case, the constant is $32768 = 2^{15}$, the total number of samples in a FFT. This would certainly result in a smaller variance, but we have not established whether this effect suffices to explain the observed disparity.

3.4.3. Repeating pulse thresholds: theory

For repeating pulses, the situation is simpler. We test a repeating pulse by adding up $M$ samples, where $M$ is very large. These samples are not all consecutive. For the minimum value of $M$, we add groups of 16 samples together, with a frequency of $F_{\text{min}} = 137$, for a
total of $M = 16 \cdot 137 = 2192$. This sum of many iid variables will have a normal distribution with variance $M$ and standard deviation $\sqrt{M}$.

The total number of potential pulses per 13 s workunit is as follows.

1. **Frequencies.** Suppose the number of samples (which is $2^{20}$ or $2^{25}$) is $n$, the dm chunk size (16 or 128) is $d$, and the sub buffer size is $\frac{n}{d} = s$. Then there are $L$ frequencies, where $(1 + \frac{1}{s})^L = 2$. That is, $e^L = 2$, so $L = s \cdot \ln(2)$.

2. **Phases.** Each frequency has a period equal to about $s/F_{\text{min}}$, and the phase of the repeating pulse can start at any point in that period.

3. **Subfrequencies.** Each frequency has a number of subfrequencies, each of which has half as many bins as the previous. This doubles the number of potential pulses.

4. **Coadds.** Each subfrequency has a number of coadds, each of which has half as many bins as the previous. This doubles the number of potential pulses again.

5. **Number of dm chunks.** The FFA runs $\frac{14208}{d}$ times for each of 2 signs.

Therefore, the total number of potential pulses is

$$s \cdot \ln(2) \cdot (s/F_{\text{min}}) \cdot 4 \cdot (14208/d) \cdot 2 = (n^2 \cdot 8 \cdot \ln(2) \cdot 14208)/(F_{\text{min}} \cdot d^3) \quad (43)$$

which is $3.09 \times 10^{11} = e^{26.5}$ for the large dm chunk, and $1.54 \times 10^{11} = e^{25.8}$ for the small dm chunk.

We want the number of standard deviations such that the probability of exceeding that number is $e^{-26.5}$ or $e^{-25.8}$, respectively. We can perform the integration in idl, finding that we need thresholds of 6.88$\sigma$ in the former case and 6.78$\sigma$ in the latter, where $\sigma = \sqrt{M}$.

### 3.4.4. Single pulse thresholds: experiment

We chose our thresholds for the single pulse search in two steps. First, we ran the client on 10 workunits, keeping track of the strongest pulses we found at each coadd. The second largest pulse out of 10 is roughly the 90th percentile, so we set our thresholds at that point. Second, we used these thresholds in the client and collected some data. After rejecting RFI, we raised thresholds post-analysis, throwing out the weaker pulses in the database.

### 3.5. Expected Sensitivity

#### 3.5.1. Sensitivity of Astropulse

To calculate Astropulse’s expected sensitivity as an area in Jansky $\cdot \mu$s, we will follow the treatments in (Rohlfs & Wilson 2000) and (Van Vleck & Middleton 1966). A signal from a telescope receiver consists of a function $f(t)$ that describes the amplitude of the signal
at time $t$. However, we will be discussing a generalization of a signal, called a “random process.” A random process is not a particular signal, but a probability distribution over the set of all signals. It is natural to talk about noise as a random process, since we do not know in advance the amplitude of the noise as a function of time. A random process can be written as $X(t)$, which is a random variable for each value of $t$, not necessarily independent. We will consider Gaussian random processes; namely, processes $X$ which can be written as:

$$X(t) = \sum_\nu A_\nu (Z_\nu + i\bar{Z}_\nu) e^{2\pi i \nu t}$$  \hspace{1cm} (44)

where $A_\nu$ are positive real constants, and $Z_\nu, \bar{Z}_\nu$ are random variables distributed independently as real Gaussians with $\sigma = 1, \mu = 0$. For such processes, the value $X(t)$ for any $t$ is distributed as a complex Gaussian with variance $\sum_\nu A_\nu^2$ in the real and imaginary components. But although $Z_\nu$ are independent, $X(t_1), X(t_2)$ may not be independent. In general, there is some correlation coefficient $r$ such that the joint pdf of the real (or imaginary) components of $X(t_1), X(t_2)$ is:

$$p(x_1, x_2) = \frac{1}{2\pi \sqrt{1-r^2}} \exp\left(\frac{1}{1-r^2}(x_1^2 - 2rx_1x_2 + x_2^2)\right)$$  \hspace{1cm} (45)

To calculate Astropulse’s sensitivity, we want to relate the following quantities. Subscript 0 refers to the signal emitted by the receiver and entering the one-bit clipper. Subscript 1 refers to the signal emitted by the one-bit clipper.

- $F =$ the signal flux entering the telescope (from an astrophysical source).
- $G =$ the gain of the telescope, in K Jy$^{-1}$.
- $kT_0 =$ the average noise power emitted by the receiver, per unit bandwidth, and entering the one-bit clipper.
- $S_0 =$ the average signal power emitted by the receiver and entering the one-bit clipper.
- $P_1 =$ the average noise power emitted the one-bit clipper, in dimensionless units.
- $S_1 =$ the average signal power emitted by the one-bit clipper, in dimensionless units.

We will assume, for simplicity, that the power $S_0$ comes from a signal with amplitude $X_0(t) = A(Z_{\nu_0} + i\bar{Z}_{\nu_0}) \exp(2\pi i \nu_0 t)$ for a single $\nu_0$. We require that $E(|X_0|^2) = 2A^2$ is the average power $S_0$, where $E$ denotes expected value. The Gaussian distribution may seem incorrect – it allows the possibility of zero signal power, for instance. But it turns out that we can calculate the precise effect of the clipper on the signal $X_0$, so we will make that calculation and then assume that the nonzero expected power is what’s important.

Next, we consider autocorrelations, and their relationship with power spectra. The autocorrelation $R_X(\tau)$ is a property of a random process $X(t)$, and has a delay parameter $\tau$. It measures the correlation between the signal at time $t$ and at time $t + \tau$. It’s defined by:
The autocorrelation is useful in several ways. The simplest is that $R_X(0) = E(|X(t)|^2)$, the expected power. Another fact is that the fourier transform of $R$ is the same as the power spectral density (PSD) $S(\nu)$. The PSD is defined as the expected value of the square of the amplitude in the frequency domain, $S(\nu) \equiv E(|\tilde{X}(\nu)|^2) = |A_\nu|^2$.

Now we compute $R_{X_0}$, the autocorrelation of $X_0$:

$$R_{X_0}(\tau) \equiv E(X_0^*(t)X_0(t+\tau))$$

$$= 2A^2\exp(2\pi i\nu_0\tau) = S_0 \exp(2\pi i\nu_0\tau)$$

Where the last step is consistent with the fact that $R_{X_0}(0)$ must equal the signal power $S_0$. Now the total power emitted by the receiver is $kT_0B + S_0$, which comes from a signal with amplitude $Y_0(t) + X_0(t)$, where $Y_0(t)$ is white noise. Since $Y_0$ and $X_0$ have a random phase relationship, the autocorrelation function of the sum is just the sum of the autocorrelation functions. (That is, $E(X_0^*(t)Y_0(t+\tau)) = E(Y_0^*(t)X_0(t+\tau)) = 0$). Then the noise is not correlated with itself over long time scales $\tau$, so $R_{Y_0}$, the autocorrelation of $Y_0$, is:

$$R_{Y_0}(\tau) = E(Y_0^*(t)Y_0(t+\tau)) = kT_0Bg(\tau)$$

For some narrow function $g(\tau)$. The total noise power is $E(|Y_0^2|) = R_{Y_0}(0)$, which must be equal to $kT_0B$, where $B$ is the bandwidth. Therefore $g(0) = 1$. So

$$R_0(\tau) = kT_0Bg(\tau) + S_0 \exp(2\pi i\nu_0\tau)$$

We need to characterize the output signal that results from clipping $R_0$. This problem is considered in (Rohlf & Wilson 2000) and (Van Vleck & Middleton 1966). In the case of a Gaussian signal, the joint pdf of amplitudes at any two times (i.e. $X_0(t_1), X_0(t_2)$) takes a closed form as above. From that form, one can derive the correlation between the probability that the clipped noise will be $\pm 1$ at $t_1$ and $t_2$. Then one can write down the autocorrelation function of the clipped noise. For the real bits alone, the autocorrelation is:

$$R_1(\tau) = \frac{2}{\pi} \arcsin\left(\frac{R_0(\tau)}{R_0(0)}\right)$$

As expected, the power is $R_1(0) = 1$. For the real and complex bits together, the autocorrelation must be twice as large, $R_1(0) = 2$. For $S_0 \ll kT_0B$, the total power is mostly due to the noise $kT_0$. However, at values $\tau \neq 0$, the contribution of $g(\tau)$ falls off quickly. Therefore,
\[ R_1(\tau) = 2 \cdot \frac{2}{\pi} \arcsin \left( \frac{S_0 \exp(2\pi i \nu_0 \tau)}{kT_0 B} \right) \]  
(54)

\[ \approx 2 \cdot \frac{2}{\pi} \frac{S_0 \exp(2\pi i \nu_0 \tau)}{kT_0 B} \]  
(55)

where we are ignoring \( R_1(0) \), because \( g(\tau) \) is narrow and we are only going to use \( R_1(\tau) \) in an integral. The power at frequency \( \nu_0 \) is found by integrating

\[ S_1(\nu) = \int R_1(\tau)e^{-2\pi i \nu \tau} d\tau = 2 \cdot \frac{2}{\pi} \delta(\nu - \nu_0) \frac{S_0}{kT_0 B} \]  
(56)

which says that the total power near \( \nu_0 \) is \( \int S_1(\nu)d\nu = 2 \cdot \frac{2}{\pi} \frac{S_0}{kT_0 B} \). Then the ratio

\[ \frac{S_{1,\text{near}(\nu_0)}}{R_1(0)} = 2 \cdot \frac{S_0}{\pi kT_0 B} \]. In other words, \( S_1/P_1 = \frac{2}{\pi} S_0/P_0 \).

As discussed above, the power has a gamma distribution, both in the frequency domain and (when dechirped) in the time domain. To summarize these results: in the absence of any input signal \( S_0 \), the frequency components' amplitudes should behave like Gaussian variables. So after a discrete Fourier transform, \( S_1(\nu_0) \) has an expected power of 2, and is distributed as a sum of the squares of two Gaussians (real and complex.) This distribution could be seen as a chi squared with 2 degrees of freedom, a gamma distribution, or an exponential distribution. In the time domain, if we dedisperse the noise by performing a Fourier transform, dividing by a chirp function (dechirping), and then performing the inverse Fourier transform, we should still end up with a sum of a real and a complex Gaussian. This should give us an exponential distribution for each time sample.

Now assume that a chirped pulse with energy \( E_0 \) will have

\[ \frac{2}{\pi} \left( \frac{E_0}{kT_0 B t_{\text{sample}}} \right) \]  
(57)

times the expected energy in one sample after the one-bit clipper. (This assumption is reasonable, since a chirped pulse looks like a monochromatic signal locally. Therefore we can apply the above reasoning, even though our proof was for monochromatic signals.) The noise will have \( N \) times the expected energy of one sample, where the duration of the pulse is \( N \) samples, so the pulse will be detectable if

\[ \frac{(2/\pi)E_0 + kT_0 B N t_{\text{sample}}}{kT_0 B t_{\text{sample}}} \]  
(58)

is above threshold for \( N \) samples, where the threshold comes from a chi squared with \( 2N \) degrees of freedom. Astropulse sets this threshold to \( H \sim 30 \) for \( N = 1 \) sample.

Then the minimum detectable energy satisfies the equation:

\[ \frac{2}{\pi} E_0 + kT_0 B N t_{\text{sample}} = kT_0 B t_{\text{sample}} H \]  
(59)
\[ \frac{2}{\pi} E_0 = kT_0 B_t \text{sample} (H - N) \quad (60) \]
\[ E_0 = \frac{\pi}{2} kT_0 B_t \text{sample} (H - N) \quad (61) \]
\[ E_0 = \frac{\pi}{2} kT_0 (H - N) \quad (62) \]

using the fact that \( B_t \text{sample} = 1 \). If a signal is detected with \( \tilde{H} \) times the energy in one bin, the implied energy of the pulse can be determined by inserting \( \tilde{H} \) into Equation 62.

We have calculated \( E_0 \), the value of the energy emitted by the receiver. This implies that the signal has a power per bandwidth of \( E_0/(BNt_\text{sample}) = E_0/N \), hence a temperature of \( T = \frac{E_0}{NT} \). This means the astrophysical source has a flux of \( T/(2G) = \frac{E_0}{2GNk} \) in this polarization. (By definition of the gain.) If it is actually unpolarized, the total flux in both polarizations is twice that, or \( T/G \). Then its flux density \( \cdot \) duration is:

\[ F \cdot (Nt_\text{sample}) = \frac{E_0 t_\text{sample}}{Gk} = \frac{\pi}{2} \frac{T_0}{Gk} (H - N) \quad (63) \]

This value could be measured in Jansky \( \cdot \) \( \mu s \), for instance. In our case:

1. \( T_0 \sim 30 \) \( K \) is the system temperature.\(^1\)
2. \( G = 10 \) \( K \) Jy\(^{-1} \) is the telescope gain, roughly equal to \( \frac{A}{k} \), where:
3. \( A = \frac{\lambda^2}{\Omega} \) is the effective area and \( k \) is Boltzmann’s constant.
4. \( \lambda = 21 \) cm is the wavelength of the signal.
5. \( \Omega = 8.3 \cdot 10^{-7} \) is the beam’s solid angle.

So the resulting flux density \( \cdot \) duration is 1.9 (\( H - N \)) Jy \( \mu s \).

3.5.2. Sensitivity comparison

While Astropulse detects a signal coherently, other undirected radio surveys use incoherent detection schemes. Typically they use a filter bank, dividing the spectrum into \( N \) sub-bands as described in section 3.2. The method amounts to measuring the deviation \( \Delta T \) from the receiver temperature \( T_0 \) at successive times, but in a different channel at each time. The channel frequency varies at a constant rate that is proportional to some dispersion measure (DM) that one is testing for.

First consider a single-channel device with bandwidth \( B \). Then following Wilson et al. (2009), we assume the spectrometer finds the power in a band by Nyquist sampling at twice the bandwidth, then finding the average of the squares. The square of a Gaussian

\(^1\)http://www.naic.edu/alfa/gen_info/info_obs.shtml
with $\sigma = 1$ is a chi squared with one degree of freedom, having a variance of 2. In this case, the power $P$ has $\mu = kT_0B$, so $P = X^2$, where $X$ is a Gaussian with $\sigma = \sqrt{kT_0B}$. Then $P/T_0B$ is a chi squared with variance 2, and $P$ has variance $2(kT_0B)^2$. The variance of the $2Bt$ samples in time $t$ is $2(kT_0B)^2(2Bt)$. Assuming we can detect a deviation of $m\sigma$, that’s $m\sqrt{2kT_0B\sqrt{2Bt}}$. To find the standard deviation of the average power per sample, divide by the $2Bt$ samples to get $mkT_0B/\sqrt{Bt}$. Then find the equivalent temperature of this minimum detectable signal, dividing by $kB$ to get:

$$T_{\text{eff}} = mT_0/\sqrt{Bt} \quad (64)$$

A multichannel filter spectrometer will have the same sensitivity, assuming it can add up the correct parts of the signal. This might be problematic in the cases of time-frequency uncertainty or dispersion within sub-bands, as discussed in section 3.2. We can understand these effects as widening the pulse width by a factor $t/W_i$ while preserving the pulse area, so that the effective temperature $T_{\text{eff}}$ is equal to $W_iT_{\text{int}}$ for an intrinsic temperature $T_{\text{int}}$. For instance, Deneva & Cordes (2008) gives the sensitivity formula as:

$$T_{\text{int}} = \left( \frac{t}{W_i} \right) \frac{mT_0}{\sqrt{N_{\text{pol}}Bt}} \quad (65)$$

$$t = (W^2_i + \Delta t^2_{\text{DM, ch}} + \Delta t^2_{\text{DM, err}} + \Delta t^2_{sc})^{1/2} \quad (66)$$

$t$ is the effective width of the pulse

$W_i$ is the intrinsic width of the pulse

$\Delta t_{\text{DM, ch}}$ is the dispersion within one channel, and is given by Equation 2

$\Delta t_{\text{DM, err}}$ is the error caused by looking at the wrong dispersion measure, and can be calculated by inserting $\frac{1}{2}$ the DM step into Equation 2, using the whole bandwidth as $\Delta \nu$.

$\Delta t_{sc} \propto f^{-4}$ is the error caused by scattering broadening.

This assumes that the channel bandwidth is not so narrow that we are sampling beyond the Nyquist rate. If the channel bandwidth were that narrow, there would be another contribution to the effective width; but all surveys are careful not to sample beyond the Nyquist rate.

In any event, if one cares about the pulse area in Jansky \cdot $\mu$s rather than the instantaneous flux, Equation 64 will suffice. One need only calculate the area of a single polarization $A = \frac{mT_0}{2G\sqrt{Bt}} = \frac{mT_b}{2G}\sqrt{t/B}$, and both polarizations together, $A = \frac{mT_b}{G}\sqrt{t/B}$. One must also compute $t$ from Equation 66.

The most difficult variable to quantify above is perhaps $t_{sc}$. We can estimate it using the empirical formula given in Lorimer & Kramer (2005):

$$\log t_{sc, ms} = -6.46 + 0.154(\log DM) + 1.07(\log DM)^2 - 3.86 \log \nu_{\text{GHz}} \quad (67)$$
However, this formula applies to sources in the Milky Way. Astropulse spends a substantial fraction of the time looking outside our Galaxy, in which case $t_{sc}$ should be much smaller, even for large DMs. But the distribution of the intergalactic medium is not well understood. Lovell et al. (2007) find that extragalactic radio sources at redshifts greater than $z = 2$ do not have microarcsecond structure, suggesting that they are scatter broadened by turbulence in the IGM. A microarcsecond of angular broadening at $z = 2$ corresponds to a pulse width of $\tau = \theta^2 d/c = 8 \mu s$ (using $d = 3.57$ Gpc). According to Ioka (2003), this is a DM of about 2000 pc cm$^{-3}$. So perhaps we can assume that at the (smaller) DMs of our experiment, pulses will have reasonably small widths. For instance, inside the galaxy, a DM of 830 pc cm$^{-3}$ would have a scattering width 400 times smaller than a DM of 2000 pc cm$^{-3}$. Even if the scattering width were nearly 8 $\mu s$, Astropulse is still good at detecting such pulses. (The threshold is just twice as high as for 1 sample pulses.) Therefore, we will ignore the scattering error in the tables below, and set $t = t_{sample}$.

In the tables, the following conventions are observed:

- $t$: the effective duration of the pulse after dedispersion
- $t_{sample}$: the time resolution
- width: the telescope beam with, in steradians
- beams: number of simultaneous beams
- $d_{\text{max}}$: the minimum distance from which an exploding $M = 10^8$ kg black hole would be visible, in kpc. It’s calculated using $U_{\text{min}} = \text{energy} / (\text{area} \cdot \text{bandwidth}) = (Mc^2)/(4\pi d_{\text{max}}^2 \cdot 1\text{GHz})$, where $U_{\text{min}}$ is the minimum detectable signal in Jy $\mu s$.
- rate: the minimum rate of black hole explosions under which such a black hole would be detectable, $V^{-1} t_{\text{obs}}^{-1}$. Here, $V = (4\pi/3)d^3n_{\text{beams}} \Omega/4\pi = \frac{1}{3}\Omega d^3 n_{\text{beams}}$ is the volume of space observed at any one time.
Table 1: Survey parameters

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\(^a\) O’Sullivan et al. (1978)
\(^b\) Phinney & Taylor (1979)
\(^c\) Amy et al. (1989)
\(^d\) Katz et al. (2003)
\(^e\) McLaughlin et al. (2006)
\(^f\) Manchester et al. (2001)
\(^g\) Lorimer & Bailes (2007)
\(^h\) Deneva & Cordes (2008)
Table 2: Survey parameters

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<td>1440</td>
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Table 3: Survey parameters

<table>
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<tr>
<th>#</th>
<th>author</th>
<th>freq (MHz)</th>
<th>G (K Jy$^{-1}$)</th>
<th>width</th>
<th>beams</th>
<th>$t_{\text{obs}}(h)$</th>
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</thead>
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<tr>
<td>1</td>
<td>O’Sullivan et al.</td>
<td>100</td>
<td>0.1</td>
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<td>46</td>
</tr>
<tr>
<td>2</td>
<td>Phinney &amp; Taylor</td>
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<td>27</td>
<td>$6.6 \cdot 10^{-6}$</td>
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<td>3</td>
<td>Amy et al.</td>
<td>3</td>
<td>-</td>
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<td>4000</td>
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<td>4</td>
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<td>$1.3 \cdot 10^{-5}$</td>
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<tr>
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Table 4: Survey parameters

<table>
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<th>#</th>
<th>author</th>
<th>$N_{\text{pol}}$</th>
<th>sens (Jy μs)</th>
<th>$d_{\text{max}}$ (kpc)</th>
<th>rate (pc$^{-3}$yr$^{-1}$)</th>
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<td>1</td>
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</tbody>
</table>

$^a$MOST has 1 mJy of noise in each beam after 12 hours, http://www.physics.usyd.edu.au/sifa/Main/MOST
4. Distributed computing: the BOINC platform

5. RFI mitigation

6. Testing and Verification

7. Results and interpretation

8. Stardust@home

9. Suggestions for further research
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